A probabilistic model to quantify the results of the research on the Turin Shroud

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Abstract

To synthesize the variety of the contrasting results coming from the researches done on the Turin Shroud, a probabilistic model capable of consider mutually excluding events, is proposed.
It is based on the definition of three possible alternatives (A=authentic, F=fake, N=not A or F) regarding the cloth origin. Each alternative is then evaluated from the assignment of the corresponding probability, with relative uncertainty, to each statement, relative to the results of the Shroud investigations.
Each statement is composed by 7 coefficients: 3 relative to the corresponding alternative (A, F and N) probability, 3 relative to the corresponding uncertainty and one to evaluate the result importance.
As an example, results obtained from a very simple probabilistic analysis that considers only 7 statements are compared with that published by other authors.

Abbreviations: ST = Shroud of Turin; MST = Man of the Shroud of Turin.

1) Introduction

The necessity to make a probabilistic model to value the results of the research carried out on the ST originated from the difficulty to succeed in judging in a global and objective way the remarkable quantity of evidences or statements brought in favour or against a thesis of authenticity or falsity.
Many scholars nowadays keep on backing up diametrically opposed[1, 2, 3] theses bringing in favour only a limited number of statements.
Once having made an interpretative model capable to analyze many statements, each scholar should be able to express more objective judgements.
In this work we have foreseen to analyze the 100 statements considered more significant, both in favour and against a particular thesis. Since some of them are scientifically verified, others are more uncertain, it raises the necessity to weigh each statement with reference to its reliability degree.
On the other hand, from the several researches carried out on a world level, a sole and complete and significant statement as to show the ST authenticity or falsity didn't come out.
The authors' opinion is that maybe the most significant evidence shall come out from the global analysis of all the results obtained till now.

(*) For merely Italian academic purposes the individual contribution of the single authors is specified as follows: G. Fanti (80%) has conceived and developed the probabilistic model, E. Marinelli (20%) has proposed and verified the reliability of the alternatives.

In the past analyses of this kind were already carried out[4, 5]. But such analyses, limited to a more reduced number of statements (five or seven), don't consider the conditioned probabilities[1]. These results are checked in the light of the new proposed formulation.

2) General considerations on the probabilistic model

One uses the inferential or inductive statistic that concerns the control of statistic hypotheses and of the valuation of quantities not previously known. It is necessary that the hypothesis to check has a statistic nature. The valuation of the unknown quantity is realized applying the Bayes formula (see Appendix 1 and § 3.3). One can say that the final probability of a hypothesis, subordinately to the obtained result, is proportional to the product of the initial probability multiplied by the likelihood of the result subordinately to the hypothesis, being the proportionality constant the probabilities of the result.
One considers like sample space[2] the whole of all the shrouds, included that one that wrapped Jesus; such a whole comprehends beside the ST also the false ones and the existent ones, or no more available because destroyed, that have wrapped human bodies.
From the sample space one considers the event E consisting in the extraction of the ST; main purpose of this work is not to value the corresponding probability of a priori extraction, but to value the a posteriori probability that the ST has owed to a particular man, Jesus, on the basis of information obtained through research carried out on the cloth.
To do this it is necessary to define first the possible alternatives of the event and subsequently, through the analysis of the research results, to define a degree of a posteriori probability for each alternative.

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[1] For instance in the throw of a die with 6 faces, for each throw there are 6 alternatives corresponding to the number of the face resulting from the throw. If the die has no defects, the alternative corresponding to number 1 has a probability out of 6 to happen; the other alternatives have the same probability.
If one throws the die twice there are (6x6=) 36 different alternatives: for instance the alternative that comes out "1" the first time and "6" the second has a probability out of (6x6=) 36 to happen. Even the alternative that comes out "1" the first and the second time has a probability out of 36 to happen.
In such a case the events are not mutually excluding because also the "mixed cases" are admitted and the resulting probability is given by the product of the probabilities of each alternative.
On the contrary, in the case of the probabilistic calculation of the ST one cannot consider a mixed alternative partly favourable, partly unfavourable to a particular thesis. On the analogy of the double throw of the die, it's like accepting the only coinciding alternatives, that is "1,1", "2,2", etc.: it is a matter of probabilities conditioned by the previous event. In such a case the probability that comes out "1" both the first and the second time is no more 1/36, but 1/6 (eq. A1.5) because the mixed cases "1,2", "1,3", .., "6,1", .., "6,5" are excluded from the analysis.
[2] To better clarify the definition of some probabilistic and of inferential statistic concepts, an experiment that explains from analogy the logic proceeding applied to the ST is quoted. It refers to a pack of 52 "bridge" cards previously "marked" by a cheat and to the distribution to a player of 13 playing-cards.

a) The sample space is the whole of all the possible hands that can be distributed to a player and they are in all $\binom{52}{13} = 6.35 \cdot 10^{11}$
b) The event consists in having taken out from the sample space that particular pack of 13 cards.
c) One can define for instance the following 3 mutually excluding alternatives that could be important for the player: A = "to have in one's hand 4 kings", F = "to have in one's hand 4 aces", N = "to have in one's hand neither 4 kings nor 4 aces".

In this work it is not only of interest the valuation of the a priori probability to have in one's hand one of the 3 alternatives, but also in the valuation by the cheat of the probabilities that his adversary has in his hand one of the three alternatives from the knowledge of some details. For instance the cheat knows that (1) "all the court-cards have a red mark", (2) "all the cards with a odd value have a green mark", (3) the knave, the king and the ace of hearts have a blue mark", etc.
One supposes then that the cheat succeeds in reading the marks colors of all the cards of his adversary. From these data one has to value the a posteriori probability that the adversary has in his hand the alternative A, F or N.

The development of the model is based on the subjectivistic conception of statistic, according to which the probability of an event is defined like the measure of the confidence degree that a consistent[3] individual attributes to the happening of the event itself. It let us weigh the different assigned coefficients as to the reliability of the statement.
The valuation of the different coefficients will be done by the single individual according to the subjectivistic formulation of the probability considered like measure of the confidence degree attributed to the happening of a certain alternative. The model is then completed by a valuation of the assigned parameters uncertainty.
There are different perplexities on the acceptation of a subjectivistic attitude as to scientific problems and such perplexities increase whenever they are problems with implications of religious kind that could affect in a not negligible way the individual's judgement that in such case risks to be little coherent.
According to the authors, a subjective judgement of the assignment however will be able to be sufficiently significant if the final result is obtained from the synthesis of the results expressed by many researchers, also having different ways of thinking.
For this reason the research develops in three different phases: the first one, object of this work, consists in the formulation of an interpretative model of probabilistic kind for the evidences and the statements; the second one regards the synthesis, in a hundred statements, of the numberless significant analyses, more or less scientific, carried out on the ST; the third one, practicable by everybody interested in, consists in assigning to each statement the opportune parameters asked by the model and reaching in this way different results. Then, it should be done a final synthesis of the provided results.

3) Probabilistic model formulation

3.1) Definition of the alternatives.

Three different alternatives that caused the event E are here considered; they include the most plausible hypotheses regarding the origin of the ST; they are possible, but excluding each other and called A, F and N:

-1) Alternative **A**: the ST is *authentic*; it wrapped Jesus' body;

-2) Alternative **F**: the ST is *false, medieval or post-medieval*: it could be a painting or the work of a so-called "ingenious forger murderer";

-3) Alternative **N**: the ST is *not authentic*, but it is neither false, medieval or post-medieval; therefore the alternatives "A" and "F" are not verified; in such an alternative all the other possible causes for the formation of the ST, miracle not excluded, are included.

Obviously the model is also suitable for the analysis of only 2 alternatives, for instance " the ST is or not a painting"; in such a case one should consider null all the coefficients relative to the third alternative.

3.2) Assignment of the probabilistic values

To the alternative A one assigns an "a priori" probability $P^I(A)$ and an absolute uncertainty i^I_A, to the alternative F an "a priori" probability $P^I(F)$ and an absolute uncertainty i^I_F, to the alternative N an "a priori" probability $P^I(N)$ and an absolute uncertainty i^I_N.

[3] For instance the probability of an event could be intended like the stake that a consistent individual is ready to pay for receiving the amount of a unit if the event happens. One speaks about a consistent individual in the sense that he must accept the chosen stake both as player and as bank.

In another work[6] the following a priori probabilities with their corresponding uncertainties are assigned and discussed:

$P^I(A)=0.05$; $i_A^I=0.02$; $P^I(F)=0.35$; $i_F^I=0.05$; $P^I(N)=0.60$; $i_N^I=0.05$

To each $_j$ statement one has to assign the following parameters:

- a_j = probability that the alternative A happens in consequence of the considered statement,
- f_j = probability that the alternative F happens in consequence of the considered statement,
- n_j = probability that the alternative N happens in consequence of the considered statement,
- i_{aj} = absolute uncertainty assigned to the probability a_j of the considered statement,
- i_{fj} = absolute uncertainty assigned to the probability f_j of the considered statement,
- i_{nj} = absolute uncertainty assigned to the probability n_j of the considered statement,
- p_j = weight assigned to the considered statement.

The weight p_j can be valued on the basis of different factors bound to the scientificity of the statement, to its significativity and to the possible dependence on other statements:

$$p_j = p_{1j}\, p_{2j}\, p_{3j} \qquad (1)$$

being:

-p_{1j}: weight bound to the *scientificity* of the statement; one could use a scheme like:
  - statement published on an international magazine: p_{1j} =1
  - published statement: p_{1j} =0.8
  - little documented statement or not verified hypothesis: p_{1j} =0.1
  - statement having a relative uncertainty i_{aj}/a_j , i_{fj}/f_j o i_{nj}/n_j over 50%: p_{1j} =0;

-p_{2j}: weight bound to the *importance* of the statement according to the scheme:
  - statement of extreme interest: p_{2j} =3
  - interesting statement: p_{2j} =2
  - standard statement: p_{2j} =1
  - statement of scarce interest: p_{2j} =0.5;

-$p_{3j,}$ effect of a possible *correlation* degree with other statements having values included between 0 and 1. For instance if there were two identical statements one should assign to each of them the value p_{5j} =0.5.

The statements subjected to analysis are by simplifying supposition among them mutually independent; the possible dependence that could be verified will be considered through the weight p that is proposed in this work to be able to compare among them statements with different importance.
In [6] the statements are subdivided in subjects to be more easily able to assign such values. For instance the not believer could assign p_{3j}=0 to the statements regarding the Old and New Testament if he doesn't consider them significant. On the contrary the believer will assign values of p_{3j} included between 0,5 and 1 to the same statements because in some of them one could find a correlation with the definition of the alternative A or with other considered statements.
The case p_j=1 has no influence on the product (eq. 3a,b,c) and therefore a typical statement must have such a coefficient.
The case p_j=2 squares the probabilistic coefficient and therefore is like considering the same statement twice in the analysis; it should be applied therefore to statements with high significacy.
In the case p_j=0,5 one calculates the square root of the coefficient a_j, f_j, n_j; therefore we have that the statement is equivalent to half typical statement.
According to the principle of the total probabilities (or of the addition, eq. A1.3), for each $_j$ statement one has to verify the condition relative to the combination of mutually excluding alternatives:

$$a_j + f_j + n_j = 1 \qquad (2)$$

that is the probabilities assigned for a certain statement should be in total the 100%.

3.3) Model construction.

It comes to accepting or not an alternative among the possible ones excluding one another. The probabilistic criterion of acceptance bases on the analysis of the statements obtained by various experiments or studies carried on the ST.
The analysis result consists in assigning probability values to the alternatives developing the following points:

- -a) PHASE I. One assigns the a priori probabilities $P^I(A)$, $P^I(F)$, $P^I(N)$, of the three alternatives completely ignoring the results of the realized research.

- -b) PHASE II. One calculates the a posteriori probabilities $P^{II}(A)$, $P^{II}(F)$, $P^{II}(N)$ or subordinate probabilities, indicated below as $P^{II}(I)$, being the index I correspondent to the three alternatives A, F, N and P(E) the probability of the event E subordinate to the happening of the alternative I.
  We give m statements to value through the assignment of the coefficients aj, f_j, n_j, p_j (with index j variable from 1 to m). The probabilities $P^{II}(I)$ come out from the composition of the single assigned probabilities (a_j, f_j, n_j), according to the principle of the composed probabilities (see eq. A1.4):

$$P^{II}(A) = \prod_{j=1}^{m} a_j^{p_j} \;;\; P^{II}(F) = \prod_{j=1}^{m} f_j^{p_j} \;;\; P^{II}(N) = \prod_{j=1}^{m} n_j^{p_j} \qquad (3a, b, c)$$

  Considering the weights p_j assigned to the various statements, the effective number m_1 of considered cases, generally different from m, becomes therefore:

$$m_1 = \sum_{j=1}^{m} p_j \, . \qquad (4)$$

  It's desirable that the condition that m_1 is equal to m, number of proofs, is verified; in such a way one can do a probabilistic reference directed to the number m of considered proofs.

  One calculates the final probabilities $P^*(I)$ without the Bayes constant:

$$P^*(I) = P^I(I)\, P^{II}(I), \quad I=A, F, N \qquad (5)$$

  One calculates the probabilities P(E) of the event E given the three alternatives resulted, according to the principle of the total probabilities (see eq. A1.3):

$$P(E) = \Sigma_{(I=A,F,N)} P^*(I) = P^I(A)\, P^{II}(A) + P^I(F)\, P^{II}(F) + P^I(N)\, P^{II}(N) \qquad (6)$$

- -c) PHASE III. Being possible the alternatives A, F, N of the considered event, but among them excluding, the mixed possibilities have to be rejected. Therefore one calculates the final probabilities $P^{III}(A)$, $P^{III}(F)$, $P^{III}(N)$ on the basis of the application of the Bayes formula:

$$P^{III}(I) = \frac{P^{I}(I)\quad P^{II}(I)}{P(E)} \quad , \qquad I=A,\ F,\ N \qquad (7)$$

4) Uncertainty analysis

Nonetheless the subjectivism of the probabilistic formulation, one considers necessary to valuate the size of the uncertainty of the result with reference to [7] supposing distributions of rectangular probabilities of the uncertainty, converted in equivalent gaussians dividing the values by 1.7. They have to be intended, where not differently specified, absolute uncertainties.
One assigns the uncertainties i_A^{I} , i_F^{I} , i_N^{I} of the a priori probabilities and the uncertainties i_{aj}, i_{fj}, i_{nj}, (j =1, m) of the probabilities of the m considered statements.
Purpose of the analysis is the determination of the uncertainties i_A^{III} , i_F^{III} , i_N^{III} of the final probabilities.
One assumes the simplificative hypothesis that the correlation degree is null and that therefore the mixed components of uncertainty don’t appear in the propagation formulas.

<u>4.1 Practical rules for the model application</u>

Here follows some proposed criteria to obtain reliable results.

-1) If the final probabilities $P^{III}(A)$, $P^{III}(F)$, $P^{III}(N)$, were affected by a too high uncertainty, it is necessary that for each statement the relative uncertainty $i_I/P(I)$ is inferior to a prefixed value. The limit relative uncertainty in this work is assumed[4] to be equivalent to 50%.
If the uncertainty exceeds the limit value one assumes that the statement is little significant and rejects it assigning a weight p=0.

-2) Since no statement, referring to the carried out research[6], is able to show with absolute certainty one of the alternatives (in the opposite case it would be useless to apply the probabilistic model proposed!), it becomes necessary that the minimum value assigned to the alternatives a_j, f_j, n_j is higher or equal to 0.0001. This is the same as supposing not to have so certain statements as to assign better probabilities than 1 out of 10000.

-3) One obtains (eq. 2):

$$a_j = 1 - f_j - n_j; \qquad f_j = 1 - a_j - n_j; \qquad n_j = 1 - a_j - f_j \qquad (8a,\ b,\ c)$$

and therefore, once assigned two absolute uncertainties, one values the third according to one of the relations deriving from the application of the eq. (A2.10):

$$i_{aj} = \sqrt{i_{fj}^{2} + i_{nj}^{2}}\ ; \quad i_{fj} = \sqrt{i_{aj}^{2} + i_{nj}^{2}}\ ; \quad i_{nj} = \sqrt{i_{aj}^{2} + i_{fj}^{2}} \qquad (9a,b,c)$$

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[4]The limit relative uncertainty is function both of the statements number object of the analysis and of the result (eq. 7) because probabilistic results percentually very different allow in some cases to accept also uncertainties relatively high. To define the limit uncertainty i_l, one can assume a relation like:

$$i_1 = \frac{K}{\sqrt{m}}\%$$

being m the number of statements considered and K a chosen coefficient equivalent to 500 in this work.

4.2) Propagation of the uncertainty to the result.

To determine the uncertainties i_A^{II} , i_F^{II} , i_N^{II} of the probabilities $P^{II}(I)$, one applies the eq. (A2.10) to the equations (3a,b,c) and obtains:

$$i_A{}^{II} = \left(\prod_{j=1}^{m} a_j^{pj}\right)\sqrt{\sum_{j=1}^{m}\left[\left(p_j \frac{i_{a_j}}{a_j}\right)\right]^2} \qquad (10a)$$

$$i_F{}^{II} = \left(\prod_{j=1}^{m} f_j^{pj}\right)\sqrt{\sum_{j=1}^{m}\left[\left(p_j \frac{i_{f_j}}{f_j}\right)\right]^2} \qquad (10b)$$

$$i_N{}^{II} = \left(\prod_{j=1}^{m} n_j^{pj}\right)\sqrt{\sum_{j=1}^{m}\left[\left(p_j \frac{i_{n_j}}{n_j}\right)\right]^2} \qquad (10c)$$

To determine the uncertainties i_A^{III} , i_F^{III} , i_N^{III} of the final probabilities $P^{III}(I)$, one applies the eq. (A2. 10) to the eq. (7) and obtains:

$$i_A{}^{III} = P^{III}(A)\sqrt{\left(\frac{i_A{}^{I}}{P^{I}(A)}\right)^2 + \left(\frac{i_A{}^{II}}{P^{II}(A)}\right)^2 + \left(\frac{i_{P(E)}}{P(E)}\right)^2} \qquad (11a)$$

$$i_F{}^{III} = P^{III}(F)\sqrt{\left(\frac{i_F{}^{I}}{P^{I}(F)}\right)^2 + \left(\frac{i_F{}^{II}}{P^{II}(F)}\right)^2 + \left(\frac{i_{P(E)}}{P(E)}\right)^2} \qquad (11b)$$

$$i_N{}^{III} = P^{III}(N)\sqrt{\left(\frac{i_N{}^{I}}{P^{I}(N)}\right)^2 + \left(\frac{i_N{}^{II}}{P^{II}(N)}\right)^2 + \left(\frac{i_{P(E)}}{P(E)}\right)^2} \qquad (11c)$$

being $i_{P(E)}$ the uncertainty on the probability of the event E:

$$i_{P(E)} = \sqrt{(P^{I}(A)\cdot i^{II}{}_{A})^2 + (P^{II}(A)\cdot i^{I}{}_{A})^2 + (P^{I}(F)\cdot i^{II}{}_{F})^2 + (P^{II}(F)\cdot i^{I}{}_{F})^2 + (P^{I}(N)\cdot i^{II}{}_{N})^2 + (P^{II}(N)\cdot i^{I}{}_{N})^2} \qquad (12)$$

Extending to the final probabilities the eq. (2), we have:

$$P^{III}(A) + P^{III}(F) + P^{III}(N) = 1 \qquad (13)$$

and therefore:

$$P^{III}(A) = 1 - P^{III}(F) - P^{III}(N); \quad P^{III}(F) = 1 - P^{III}(N) - P^{III}(A); \quad P^{III}(N) = 1 - P^{III}(A) - P^{III}(F) \qquad (14)$$

Once assigned two absolute uncertainties one can value the third according to one of the relations deriving from the application of the eq. (A2. 10):

$$i_a{}^{III} = \sqrt{\left(i_f{}^{III}\right)^2 + \left(i_n{}^{III}\right)^2}\ ; \quad i_f{}^{III} = \sqrt{\left(i_n{}^{III}\right)^2 + \left(i_a{}^{III}\right)^2}\ ; \quad i_n{}^{III} = \sqrt{\left(i_a{}^{III}\right)^2 + \left(i_f{}^{III}\right)^2} \qquad (15a,b,c)$$

5) Results

Like example, one applies the proposed model to the statements discussed in [4]: 7 statements are considered supposing the existence of only two alternatives:

- alternative A: the ST is authentic;
- alternative F: the ST is false,

to those the author could have implicitly assigned the following a priori probabilities:

- $P^I(A)=0.5$; $P^I(F)=0.5$.

Not being considered the alternative N and the weight p, we have:

- $P^I(N) = 0$; $P^{II}(N) = 0$; $P^{III}(N) = 0$; $p_j=1$;

moreover it is not considered the uncertainty in the first phase of the example; in the second phase the authors will assign as an example some indicative values.

5.1 Applicative example part I

The statements object of the analysis are the following:

1) The MST had a cloth [a_1=0.667, f_1=0.333];
2) The MST stayed little in the cloth [a_2=0.95, f_2=0.05];
3) The blood of the MST results to be perfectly separated from the cloth without blurs or singeings [a_3=0.98, f_3=0.02];
4) The MST was fixed to the cross with nails [a_2=0.333, f_2=0.667];
5) On the MST appears wounds due to a helmet of thorns: [a_2=0.999, f_2=0.001];
6) The MST received a blow of lance to the right side [a_2=0.80, f_2=0.20];
7) The MST's face has a stately and sad appearance, and at the same time, nobly serene [a_2=0.9999, f_2=0.0001];

Applying the eq. (3a,b), one obtains the following probabilities $P^{II}(I)$:

- $P^{II}(A) = 0.1653$; $P^{II}(F) = 4.444 \cdot 10^{-12} = 1 / 225\,000\,000\,000$

From the eq. (6) one obtains P(E):

- $P(E) = P^I(A)\, P^{II}(A) + P^I(F)\, P^{II}(F) = 0.08266$

From the eq. (7) one obtains the final probabilities $P^{III}(A)$, $P^{III}(F)$:

- $P^{III}(A) = 0.9999...$; $P^{III}(F) = 5.376 \cdot 10^{-11} = 1 / 18\,600\,000\,000$

Instead of a probability out of 225 thousand millions[4] it would result therefore a probability out of 18 thousand millions ca.[5]

5.2 Applicative example part II

We assume[6]: $P^I(A)=0.05$; $i_A{}^I=0.02$; $P^I(F)=0.95$; $i_F{}^I=0.02$.

[5] If it were not considered the probabilities $P^I(I)$, we would obtain the following final probabilities:
$P^{III}(A) = 0.9999...$; $P^{III}(F) = 2.688 \cdot 10^{-11} = 1 / 37\,200\,000\,000$.

From the eq. (9a,b), it results $i_a=i_f=i$; we assign therefore the following uncertainties:

1) i=0,1; 2) i=0,02; 3) i=0,01; 4) i=0,1; 5) i=0,0003; 6) i=0,08; 7) i=0,00003

Therefore it results:

$$P^{II}(A) = 0.165; \quad P^{II}(F) = 4.44 \cdot 10^{-12}; \quad m1=7; \quad P(E) = 0.00826;$$
$$P^{III}(A) = 0.99999...; \quad P^{III}(F) = 5.11 \cdot 10^{-10}=1/1\,958\,000\,000; \quad i = 5.5 \cdot 10^{-10}$$

Therefore it results that the probability that the ST is false is:

$$P^{III}(F) = 5 \cdot 10^{-10} \pm 5 \cdot 10^{-10}$$

equivalent to about 1 / 2 000 000 000 with an uncertainty of 100%. The high relative uncertainty of the result is due to the uncertainties, percentually of high entity, that were assigned.
The result $P^{III}(F)$ can therefore vary, with a confidence level of 95%, between 0 and 1/1 000 000 000.
According to the considered statements and to the probabilities with assigned relative uncertainties, this means that the probability that the ST is false, instead of being equivalent to one out of 225 thousand millions[4], wavers between zero and a probability out of one thousand millions.

6) Discussion

At this point a doubt can rise on the physical meaning of the proposed probabilistic research's result; the doubt regards the interpretation of the result in terms of acceptation or not of a possible alternative. One can bring back the problem to the definition of a limit probability beyond which the result is not scientifically acceptable.
Referring to a practical example, one considers the following statement "Every year a cube of granite with a side of 1 m, not pressed by exterior forces, rises of at least one meter from the ground".
Anybody is sure that such statement is absolutely false because it describes a physically impossible phenomenon. However this wouldn't result so clearly from a detailed probabilistic research.
One should consider the probabilistic combination of all the forces caused by the accidental collisions of the molecules forming both the block of granite and the surrounding environment. The calculus of probability would provide a result possible with a probability of one divided by a number followed by some tens of thousand millions of zeroes.
If from a theoretical point of view the result is extremely unlikely but possible, on the contrary from a practical point of view the result has to be considered really impossible.
It is not yet completely defined which could be the limit probability beyond which consider "impossible" a physical phenomenon.
One can think of the example of the roulette game. The probability that number 36 comes out after a stake is 1/37 (all the possibilities included between zero and 36 do exist); the probability that number 36 comes consecutively out twice running is $1/37^2=1/1369$; the probability that number 36 comes consecutively out one hundred times running is $1/37^{100}$ that is one divided a number followed by 156 zeroes.

According to the authors an alternative that has a probability of $1/10^{30}$ to happen could be defined "impossible". In fact this would correspond to the probability to obtain 19 times running[6] the coming out of number 36 at the roulette.
According to this formulation the probability $P^{III}(F)$, variable between zero and 1 out of one thousand millions provides the following information: even if it is very unlikely that the ST is false, such alternative is not to be excluded. However it would be more probable that number 36 comes consecutively out 5 times running at the roulette game than the ST is false.

7) Conclusions

The probabilistic model we propose, through mutually excluding events, is able to synthesize the numerous and also contrasting results of the research on the ST.
It basis on the definition of three possible alternatives (A= true, F= fake, N= not authentic) for the origin of the cloth and on the valuation of each alternative on the basis of the assignment of opportune probabilities, with relative uncertainty, to each result of the researches carried out.
The considered applicative example compares the results obtained by a probabilistic analysis carried out on only 7 statements, with those already elaborated in a more over-simplified way in bibliography pointing out that the alternative (F) can vary, with a confidence level of 95%, between 0 and 1/1 000 000 000. The probability that the ST is false, according to the considered statements, instead of being one out of 225 thousand millions, like proposed in bibliography[4], wavers therefore between zero and a probability out of one thousand millions.
If all the conditions of applicability are respected, the model could be used to synthesize a very higher number of scientific results (even 100): this is the object of another research[6].

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9) D. Costantini: "Introduzione alla probabilità" Boringhieri, Torino 1977.

[6]To obtain the result one applies the relation :

$$n = [\log_{10} 10^{30}] \ / \ [\log_{10} (1/37)] = 30 / 1,568 = 19$$

being n the number of consecutive times that number 36 comes out.

APPENDIX 1: Recalls of probabilities and of statistic valutation

Considering any group Ω of elements ω; the group Ω is called *sample space* and the elements ω *sample points*.

Accidental event is a group E of sample points and therefore a subgroup of the sample space Ω. The group formed by only a sample point is called *simple*, if more points form it, then is called *compound*.

One reminds from the algebra of the events that, given two events E_1 and E_2, the event $E_1 \cap E_2$ (intersection) happens if and only if E_1 and E_2 happen jointly, while the event $E_1 \cup E_2$ (union) happens if and only if at least one of the two events E_1 and E_2 happens.

An event is called *total* if it can be realized in different ways excluding one another and has as *probability the sum of the probabilities of the various alternatives*.

One calls measure of probability or *distribution of probability* on a group Ω a function P defined for all the events of Ω with the following properties:

-a) to each event is associated a real number not negative P(E) called probability of E

-b) if one has 2 events E_1 and E_2 of Ω mutually excluding, such as $P(E_1 \cap E_2) = 0$, one has:

$$P(E_1 \cup E_2) = P(E_1) + P(E_1) \tag{A1.1}$$

-c) the probability that Ω happens is equivalent to one: $P(\Omega)=1$.

Two events A and B are called *independent* if the intersection probability of A and B is equal to the product[7] of A and B's probabilities:

$$P(A \cap B) = P(A)\, P(B) \tag{A1.2}$$

They are considered independent events in the sense that the various sample points considered are not correlated among them.

The *total probability* that at least one excluding or mutually incompatible events happens, is equivalent to the sum of the probabilities of the single events (principle of the total probabilities or of the addition). The probability of the union event of n mutually excluding events is the sum of the probabilities[8] of the single events,

$$P(E) = \sum_{i=1}^{n} P(E_i) \tag{A1.3}$$

The *compound probability* that more mutually compatible events happen is equivalent to the product of the probabilities of the single events (principle of the compound probabilities or of the multiplication):

$$P(E) = \prod_{i=1}^{n} P(E_i) \tag{A1.4}$$

that is valid if the events are mutually independent; if the events are dependent, the probability of the $_j$ event is calculated considering that all the previous events have happened.

Conditioned or subordinate events happens whenever the statistic independence between two A and B events is not verified. If the event A is not independent from B, is shown by the symbol "A|B" and is read "A conditioned by B". The *probability* of the event A *conditioned* to the event B is:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \tag{A1.5}$$

that one can write, according to the principle of the compound probabilities like[9].

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[7] The independence is verified if the result of the first event doesn't condition the probability of the second. For instance, considering a die with 6 faces without defects thrown twice: the probability that the first time comes out 4 and the second comes out 6 corresponds to (1/6)(1/6)=1/36.

[8] For instance the probability that comes out 1 or 3 or 6 in a throw of a die without defects is (1/6 + 1/6 + 1/6) = 0,50.

[9] For instance in the throw of a 6 faces die without defects, the event A is the coming out of the face with 4 and the event B is the coming out of a face with an even number. The A probability is 1/6 and the B probability is 1/2. The probability A conditioned to B, is higher and results P(A|B) =1/3 since from the eq. (A1.5) one has P(B)=1/2 e $P(A \cap B)=1/6$.

$$P(A \cap B) = P(B)\,P(A|B) = P(A)\,P(B|A) \qquad (A1.6)$$

the last equality being verified if $P(A) \neq 0$.

Independent events: on the contrary if the events A and B are independent, that is if the intersection probability of A and B is equivalent to the product of the probabilities of A and B, one has:

$$P(A|B) = P(A) \qquad (A1.7)$$

that is not important the fact that B happens in order to the event A. In such case according to the *principle of the compound probabilities* one has[10]:

$$P(A \cap B) = P(A)P(B) \qquad (A1.8)$$

It comes to accept or not an alternative A, F, N among the possible a priori ones, each of them excluding one another, on the basis of the analysis of the statements obtained by various experiments and studies on the ST.

One considers the probability that the event E happens with one and only one of the three alternatives A, F, N. According to the principle of the total probabilities, the probability of the event E is:

$$P(E) = \sum_{I=A,F,N} P(E \cap I) \qquad (A1.9)$$

According to the principle of the compound probabilities, $P(E \cap I)$ is the probability that both the event E and the alternative I happen and corresponds to the equation (A1.6) having replaced I and E with A and B.

APPENDIX 2: Recalls of uncertainty analysis

Given a variable x and a dependent variable y=f(x), x is supposed to be valued a certain number of times, to establish its average value and relative repeatability ($t\ \sigma$) given by the product of the root mean square by the covering factor t that depends on the established confidence degree.

Generally one adopts the confidence degree (P= 95%) in all the calculations of the uncertainty to which corresponds a covering factor t=2 if the sample is sufficiently numerous.

The x value will be included within the interval $\bar{x} \pm t\sigma$, and one supposes that the y value falls within the interval defined by:

$$\bar{y} \pm d\,y = f(\bar{x} \pm t\sigma) \qquad (A2.1)$$

Developing the (A2.1) in Taylor series, one obtains:

[10] For instance from a case with 10 little disks of which 3 red and 7 black, one extracts two little disks without putting the first back in the case and supposes that each little disk has an equivalent probability to be extracted. One considers the following events: **A**: "at the first stroke one draws a red little disk"; **B**: "at the first and at the second stroke one draws a red little disk"; **C**: "at the second stroke one draws a red little disk". The sample space Ω consists of (10x9=90) results. A consists of (3x9=27) points because there are 3 possible choices for the first little disk and 9 for the second; B consists of (3x2=6) points. The probabilities of the events A, B, C are: **P(A)** =(3x9):(10x9) **= 0.3** ; **P(B)** = (3x2):(10x9) = 1:15 **= 0.0667**. **P(C)** =(7x3 + 3x2 because it is possible to draw or one of the 7 black little disks the first time and one of the 3 red the second time, or one of the 3 red little disks the first time and one of the 2 remained the second time):(10x9) **= 0.3** The conditioned probability of C given A, that is the probability of C supposing the happening of the event A is:

$$\mathbf{P(C\,|\,A)} = \frac{P(C \cap A)}{P(A)} = \frac{P(B)}{P(A)} = (6/90)\ /\ (27/90) = 20/90 = \mathbf{0.222}$$

On the contrary, if one puts the drawn little disk back in the case one would have independent events and therefore the probability **P'(B)** that the event C subordinate to A (and therefore the event B) happens, would be:

P'(B) = P(A) P(C) = 0.3x0.3 **= 0.09 > P(B)** = 0.0667

being P(A)=(3x10=30), P(B)=(3x3=9), P(C)=(10x3=30).

$$\bar{y} \pm d\, y = f\,(\bar{x}) \pm \left[\left(\frac{dy}{dx}\right)_{x=\bar{x}} t\sigma \;+\; \frac{1}{2}\left(\frac{d^2 y}{dx^2}\right)_{x=\bar{x}} (\,t\,\sigma)^2 + .. \right] \qquad \text{(A2.2)}$$

The mean y value will be equivalent to f ($\bar{x}$). The term between square bracket will be therefore the valuation of d y. Making a linear approximation of d y, one obtains:

$$d\, y \approx \left(\frac{dy}{dx}\right)_{x=\bar{x}} .\; t\,\sigma \qquad \text{(A2.3)}$$

where the term $\left(\frac{dy}{dx}\right)_{x=\bar{x}}$ is the index of the y sensibility to the changes of x. The uncertainty i_y of y depends on the uncertainty i_x of x, according to the relation:

$$i_y = \left(\frac{dy}{dx}\right)_{x=\bar{x}} i_x \qquad \text{(A2.4)}$$

Extending the model to multivariable relations, one considers a result q determined by the relation:

$$q = f_1\,(x_1, x_2, ..., x_L); \qquad (x_1, ..., x_L, = \text{independent variables}) \qquad \text{(A2.5)}$$

The best valuation of the measured value q’ will be:

$$q' = \bar{q} \pm i_q \;\; (P\%) \qquad \text{(A2.6)}$$

where one can get the mean value out of:

$$\bar{q} = f_1\left(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_L\right) \qquad \text{(A2.7)}$$

and the uncertainty i_q is obtained by:

$$i_q = f_2\left(i_{x1}, i_{x2}, ..., i_{xL}\right) \qquad \text{(A2.8)}$$

Once defined the sensibility index like:

$$J_i = \frac{\partial q}{\partial x_i}\Big|_{x=\bar{x}} \qquad i = 1, 2, ..., L \qquad \text{(A2.9)}$$

with reference to the changes of each x_i as for q; therefore the uncertainty contribute of x_i to the result q is valued by the term ($J_i\, i_{xi}$). The most probable valuation of i_q is given by the formula:

$$i_q = \pm \sqrt{\sum_{i=1}^{L}\left(J_i\, i_{xi}\right)^2} \qquad (P\%) \qquad \text{(A2.10)}$$